

CKM Phases from CP Asymmetries ¹

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Abstract

Measuring phases in the Cabibbo-Kobayashi-Maskawa matrix through CP asymmetries in B decays is a major goal of current and future experiments. Methods based on charge-conjugation and isospin symmetries involve very little theoretical uncertainties, while schemes based on flavor SU(3) involve uncertainties due to SU(3) breaking. Resolving these uncertainties requires further studies involving a dialogue between theory and experiments.

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1 Introduction

Two important developments in K and B physics took place in the past two years towards a better understanding of CP violation. First, *after thirty five years of search* direct CP violation was finally observed in $K^0 \rightarrow \pi\pi$ [1], confirming *qualitatively* a prediction of the Cabibbo-Kobayashi-Maskawa (CKM) picture [2]. A precise calculation of the measured effect within the CKM framework is precluded by theoretical uncertainties in evaluating hadronic matrix elements of low energy effective weak transition operators.

Second, *after one year of operation* the two asymmetric e^+e^- B -factories, the PEP-II machine at SLAC [3] and the KEK-B collider in Japan [4], demonstrated with their new detectors, BaBar and Belle, a promising discovery potential for CP asymmetry in $B^0 \rightarrow J/\psi K_S$. This followed a few earlier attempts to measure this asymmetry, by the CDF Collaboration at the Tevatron [5] involving comparable errors, and by the OPAL and ALEPH Collaborations at LEP [6] involving larger errors. The present world average value, $\sin 2\phi_1 \equiv \sin 2\beta = 0.49 \pm 0.23$ [7], extracted from all five measurements, is consistent with the CKM prediction where the asymmetry is given by $\sin 2\beta$ [8]. At the same time direct CP asymmetries were measured by the CLEO Collaboration at CESR [9] for several self-tagging hadronic and radiative B decays. Whereas experimental errors are somewhat smaller than in $B^0 \rightarrow J/\psi K_S$, CKM asymmetry predictions for these decays involve large theoretical uncertainties as in the case of $K^0 \rightarrow \pi\pi$. At this point all present direct CP measurements are consistent with zero asymmetries.

In the next few years substantial improvements in B decay asymmetry measurements are expected. Assuming the CKM framework, large nonzero asymmetries should be observed in certain decays. Of particular importance are decays such as $B^0 \rightarrow J/\psi K_S$, where the asymmetry determines a CKM weak phase *within an excellent approximation* [10]. In other cases, to be discussed in this talk, the determination of weak phases requires *measuring rates and asymmetries in several symmetry-related processes*. Relations between processes which permit phase determinations follow in the best case from accurate symmetries of strong interactions, such as charge-conjugation or isospin. In other cases, where less accurate symmetries such as flavor SU(3) must be employed, one may obtain information about symmetry breaking from other B decay processes.

The purpose of this talk is to review some of the methods suggested

in the past ten years, based on this general idea for determining the two “problematic” weak phases $\phi_2 \equiv \alpha$ and $\phi_3 \equiv \gamma$. Quite a few B and B_s decay modes were shown to be useful in this respect. We choose a few relatively simple examples to represent a much broader effort.

Charge-conjugation symmetry is used in Section 2 for two measurements of γ , while the extraction of α in Section 3 is based on isospin symmetry. We refer to these methods as accurate determinations of weak phases since they are based on good symmetries of strong interactions. In some cases these methods involve the experimental challenge of measuring rare processes with sufficient precision, which may not be achieved in the first round of experiments. Various applications of flavor SU(3) symmetry in charmless B decays are discussed in Section 4, stressing in particular the use of U-spin symmetry in determining the weak phase γ . We argue that uncertainties due to SU(3) breaking effects may be reduced, or even completely eliminated, by measuring these effects in certain processes. We conclude in Section 5.

In each of these schemes one measures a simple trigonometric function of a phase, such as the sine of twice the angle ($\sin 2\alpha$) or the square of the sine of an angle ($\sin^2 \gamma$). This leaves discrete ambiguities in the solutions for the angles themselves. Such ambiguities, which may hide new physics effects, can be resolved by other rather challenging measurements of different trigonometric functions of the phases, and will be discussed elsewhere at this conference [11]. In the presence of an ambiguity a conservative strategy will be to choose a phase value consistent with the CKM framework. This should eventually improve to a higher precision our knowledge of the CKM mixing matrix. Alternatively, if inconsistencies are found, they would provide probes for new physics.

Before starting a discussion of specific methods, we mention a more ambitious approach, discussed elsewhere at this conference [12, 13]. Hadronic B decay amplitudes into two light mesons are calculated within QCD to leading order in a heavy quark expansion in terms of weak phases and several non-perturbative quantities including form factors, light cone quark distributions in mesons, and chirality enhanced large corrections which occur formally at order $1/m_b$. This approach is comparable to the calculation of direct CP violation in $K^0 \rightarrow \pi\pi$ [2], with the disadvantage that strong phases cannot be measured as in K decays. The advantage lies in the possibility, at least in principle, of carrying out a systematic heavy quark expansion. When applied to weak phase determinations, this approach suffers from the same

uncertainties due to SU(3) breaking as discussed in Section 4. The schemes discussed in this section for controlling SU(3) breaking effects apply also to this approach.

2 Accurate determinations of γ

2.1 γ from $B \rightarrow DK$

In $B^+ \rightarrow DK^+$ two amplitudes interfere due to color-favored $\bar{b} \rightarrow \bar{c}u\bar{s}$ and color-suppressed $\bar{b} \rightarrow \bar{u}c\bar{s}$ transitions. This provides a few variants of a basically very simple idea [14] for determining the relative weak phase γ between the two amplitudes. We will describe two variants, in both of which one is trying the measure γ through this interference [15]. Let us discuss these two cases in some detail.

1. B decay to K and flavor specific D^0 modes [16]

The three-body decay $B^+ \rightarrow (K^-\pi^+)_D K^+$, where the $K^-\pi^+$ pair has a D^0 invariant mass, involves an interference between two cascade amplitudes,

$$Aa_{K\pi} \equiv A(B^+ \rightarrow D^0 K^+)A(D^0 \rightarrow K^-\pi^+) , \quad (1)$$

and

$$\bar{A}\bar{a}_{K\pi} \equiv A(B^+ \rightarrow \bar{D}^0 K^+)A(\bar{D}^0 \rightarrow K^-\pi^+) . \quad (2)$$

The first amplitude A , due to $\bar{b} \rightarrow \bar{u}c\bar{s}$, is color-suppressed and subsequently the D^0 decays into a Cabibbo-favored mode with amplitude $a_{K\pi}$. The second amplitude \bar{A} from $\bar{b} \rightarrow \bar{c}u\bar{s}$ transition is color-favored, and subsequently \bar{D}^0 decays with a doubly Cabibbo-suppressed amplitude $\bar{a}_{K\pi}$. The relative weak phase between A and \bar{A} is γ , their strong phase-difference will be denoted δ , $\text{Arg}(A/\bar{A}) = \delta + \gamma$, and the relative phase between $a_{K\pi}$ and $\bar{a}_{K\pi}$ (including a relative weak phase π) will be denoted $\Delta_{K\pi} \equiv \text{Arg}(a_{K\pi}/\bar{a}_{K\pi})$. Omitting a common phase space factor,

$$\begin{aligned} A(B^+ \rightarrow (K^-\pi^+)_D K^+) &= Aa + \bar{A}\bar{a} , \\ \Gamma(B^+ \rightarrow (K^-\pi^+)_D K^+) &= |Aa|^2 + |\bar{A}\bar{a}|^2 + 2|A\bar{A}a\bar{a}|\cos(\delta + \Delta + \gamma) , \end{aligned} \quad (3)$$

where $a \equiv a_{K\pi}$, $\bar{a} \equiv \bar{a}_{K\pi}$, $\Delta \equiv \Delta_{K\pi}$.

The rate for the charge-conjugate process, $B^- \rightarrow (K^+\pi^-)_D K^-$, has a similar expression in which γ occurs with an opposite sign, while strong phases are invariant under charge-conjugation. The CP asymmetry in this process, involving an interference of $Aa_{K\pi}$ and $\bar{A}\bar{a}_{K\pi}$, is proportional to $\sin(\delta + \Delta_{K\pi}) \sin \gamma$.

Let us summarize the present updated information on the parameters appearing in Eqs. (3). The three amplitudes \bar{A} , $a_{K\pi}$ and $\bar{a}_{K\pi}$ have already been measured [17, 18]. The measured ratio $|\bar{a}_{K\pi}/a_{K\pi}| = (1.21 \pm 0.13) \tan^2 \theta_c = 0.062 \pm 0.007$ is consistent at 90% confidence level with flavor SU(3) symmetry, which predicts a value of $\tan^2 \theta_c$ for the ratio of amplitudes [19]. The amplitude A can be estimated as follows. It involves a CKM factor of $|V_{ub}^* V_{cs}|/|V_{cb}^* V_{us}| \approx 0.4$ relative to \bar{A} , and is expected to be color-suppressed relative to this amplitude by a factor of about 0.25, measured in $B \rightarrow \bar{D}\pi$ decays [20]. Thus one estimates $|A/\bar{A}| \sim 0.1$.

Therefore, the two amplitudes interfering in Eqs. (3) are anticipated to be comparable in magnitude, $|\bar{A}\bar{a}_{K\pi}/Aa_{K\pi}| \sim 0.6$. This, and large final state phases measured in Cabibbo-favored $D \rightarrow K\pi$ decays [21], raised the hope [16] for a possible large CP asymmetry in this process. We note, however, that the relevant phase $\Delta_{K\pi}$ between $a_{K\pi}$ and $\bar{a}_{K\pi}$ vanishes in the SU(3) limit [19] and, as mentioned above, SU(3) does not seem to be strongly broken in $|\bar{a}_{K\pi}/a_{K\pi}|$. A recent study [22] suggests that $\Delta_{K\pi}$ is unlikely to be larger than about 20° .

The rate expression (3) and its charge-conjugate provide two equations for the three unknowns: A , $\delta + \Delta_{K\pi}$ and γ . To solve for γ requires observing another doubly Cabibbo-suppressed D^0 decay mode. Such a study is currently under way in the $K^+\pi^-\pi^0$ channel [23], and should soon provide a result for the doubly Cabibbo-suppressed amplitudes $\bar{a}_{K^+\rho^-}$, $\bar{a}_{K^{*+}\pi^-}$ and $\bar{a}_{K^{*0}\pi^0}$. Assuming, for instance, a knowledge of $\bar{a}_{K\rho}$, two equations identical to Eqs. (3) and its charge-conjugate can be written for $\Gamma(B^+ \rightarrow (K^-\rho^+)_D K^+)$ and $\Gamma(B^- \rightarrow (K^+\rho^-)_D K^-)$ involving $a \equiv a_{K\rho}$, $\bar{a} \equiv \bar{a}_{K\rho}$, $\Delta \equiv \Delta_{K\rho}$. This introduces in the four equations only one new unmeasurable quantity, $\delta + \Delta_{K\rho}$, such that these equations can be solved for γ modulo some discrete ambiguities. The ambiguities may be reduced by including information from other doubly Cabibbo-suppressed modes [24].

This method requires a large number of B 's, at least of order $10^8 - 10^9$. This is obvious, since, for instance, $\mathcal{B}(B^+ \rightarrow \bar{D}^0 K^+) \mathcal{B}(\bar{D}^0 \rightarrow K^-\pi^+) = (4.2 \pm 1.4) \times 10^{-8}$ [17, 18].

2. B decay to K and D^0 CP-eigenstate modes [25]

Neglecting very small CP violation in $D^0 - \bar{D}^0$ mixing, one can write neutral D meson even/odd CP states (decaying, for instance, to K^+K^- or $K_S\pi^0$) as $D_\pm^0 = (D^0 \pm \bar{D}^0)/\sqrt{2}$. Consequently, one has up to an overall phase

$$\sqrt{2}A(B^+ \rightarrow D_\pm^0 K^+) = \pm|\bar{A}| + |A| \exp[i(\delta + \gamma)] . \quad (4)$$

Let us define charge-averaged ratios of rates for positive and negative CP states relative to rates corresponding to color-favored neutral D flavor states

$$R_\pm \equiv \frac{2[\Gamma(B^+ \rightarrow D_\pm K^+) + \Gamma(B^- \rightarrow D_\pm K^-)]}{\Gamma(B^+ \rightarrow \bar{D}^0 K^+) + \Gamma(B^- \rightarrow D^0 K^-)} , \quad (5)$$

and two corresponding pseudo-asymmetries

$$\mathcal{A}_\pm \equiv \frac{\Gamma(B^+ \rightarrow D_\pm K^+) - \Gamma(B^- \rightarrow D_\pm K^-)}{\Gamma(B^+ \rightarrow \bar{D}^0 K^+) + \Gamma(B^- \rightarrow D^0 K^-)} . \quad (6)$$

These quantities do not require measuring the color-suppressed rate $\Gamma(B^+ \rightarrow D^0 K^+)$ and its charge-conjugate. One finds

$$\begin{aligned} R_\pm &= 1 + |A/\bar{A}|^2 \pm 2|A/\bar{A}| \cos \delta \cos \gamma , \\ \mathcal{A}_- &= -\mathcal{A}_+ = |A/\bar{A}| \sin \delta \sin \gamma . \end{aligned} \quad (7)$$

In principle, Eqs. (7) provide sufficient information to determine the three parameters $|A/\bar{A}|$, δ and γ , up to certain discrete ambiguities. However, as explained above, one expects $|A/\bar{A}| \sim 0.1$. Such a value would be too small to be measured with high precision from the tiny deviation from unity of $(R_1 + R_2)/2 = 1 + |A/\bar{A}|^2$.

Nevertheless, one obtains two interesting bounds

$$\sin^2 \gamma \leq R_\pm , \quad (8)$$

which could potentially imply new constraints on γ in future experiments. Assuming, for instance, $|A/\bar{A}| = 0.1$, $\delta = 0$, $\gamma = 40^\circ$, one finds $R_2 = 0.85$. With 10^8 B^+B^- pairs, and using measured B and D decay branching ratios [17], one estimates an error [25] $R_2 = 0.85 \pm 0.05$. In this case, Eq.(8) excludes the range $73^\circ < \gamma < 107^\circ$ with 90% confidence level. Including measurements of the CP asymmetries \mathcal{A}_\pm could further constrain γ . Again, this method

would require at least $10^8 - 10^9$ B 's similar to the previous scheme. In fact, both methods could and should be combined to improve precision [26].

The large number B 's needed to measure an asymmetry reflects the small color-suppressed rate $\Gamma(B^+ \rightarrow D^0 K^+) \propto |A|^2$ and the combined D^0 branching ratio into CP-eigenstates which is a few percent. Although $|A|^2$ does not have to be measured, one can show in general [27] that, whenever an asymmetry has to be measured due to an interference between two processes, the required number of events is dictated by the branching ratio of the rarer process and is independent of the more frequent process. For this reason it would be much preferable to use $B \rightarrow D_{\pm} K$ rather than $B \rightarrow D_{\pm} \pi$.

Very recently a variant of this scheme was proposed [28], in which one measures in flavor tagged B^0 decays to two vector mesons, $B^0 \rightarrow D^{*0} K^{*0}$, both the time dependence in this process and the angular dependence in $D^{*0} \rightarrow D^0 \pi^0$ and $K^{*0} \rightarrow K_S \pi^0$. Measuring interference terms between different helicity amplitudes permits a determination of $\sin^2(2\beta + \gamma)$, involving the sum of the weak phase in $B^0 - \bar{B}^0$ mixing and the phase in B decay. This would provide information on γ , assuming that by the time of this measurement β will have been determined. It is claimed that the sensitivity of this method is not limited by the small color-suppressed rates as it is in the above two schemes for measuring γ . However, flavor tagging suppresses the number of events, and a detailed angular analysis may be statistics limited.

2.2 γ from $B_s(t) \rightarrow D_s K$

The time-dependent decay rate for $B_s(t) \rightarrow D_s^- K^+$ is expected to exhibit an oscillating behavior including interference of two amplitudes, $A_s = A(B_s \rightarrow D_s^- K^+)$ and $\bar{A}_s = A(\bar{B}_s \rightarrow D_s^- K^+)$, from quark transitions $\bar{b} \rightarrow \bar{c} u \bar{s}$ and $b \rightarrow u \bar{c} s$, respectively. Both amplitudes, of order λ^3 , are color-allowed and involve a relative weak phase γ . The interference between the two amplitudes leads to a $\sin(\Delta m_s t)$ term in the rate, as in decays to CP-eigenstates. A major experimental challenge will be to measure precisely the high oscillation frequency. For simplicity, let us first neglect the width-difference between the two strange B meson mass-eigenstates. Denoting the relative strong phase between A_s and \bar{A}_s by δ_s , one obtains the well-known result for the general time-dependence of neutral B decays [29]

$$\Gamma(B_s(t) \rightarrow D_s^- K^+) = e^{-\Gamma_s t} [|A_s|^2 \cos^2\left(\frac{\Delta m_s t}{2}\right) + |\bar{A}_s|^2 \sin^2\left(\frac{\Delta m_s t}{2}\right)]$$

$$+ |A_s \bar{A}_s| \sin(\delta_s + \gamma) \sin(\Delta m_s t)] . \quad (9)$$

Due to the invariance of strong phases under charge-conjugation, the same strong phase δ_s occurs also in decay rates for charge-conjugate initial and final states, $\bar{B}_s(t) \rightarrow D_s^- K^+$, $B_s(t) \rightarrow D_s^+ K^-$, $\bar{B}_s(t) \rightarrow D_s^+ K^-$. The weak phase changes sign under charge-conjugation. Thus, all four time-dependent rates can be expressed in terms of four quantities, $|A_s|$, $|\bar{A}_s|$, $\sin(\delta_s + \gamma)$ and $\sin(\delta_s - \gamma)$. Measuring the time-dependence of these four processes, all of which require flavor tagging of the initial strange B meson, permits a determination of γ up to a discrete ambiguity [30].

A more precise expression than (9) includes a dependence on the width-difference $(\Gamma_L - \Gamma_H)/\Gamma_{\text{ave}}$, which is expected to be of order 10–20 %. Assuming that the two exponential decays due to two different lifetimes can be separated by this measurement, one obtains useful information also from untagged rates [31]:

$$\begin{aligned} \Gamma(B_s(t) \rightarrow D_s^- K^+) + \Gamma(\bar{B}_s(t) \rightarrow D_s^- K^+) = \\ \frac{1}{2}(|A_s|^2 + |\bar{A}_s|^2)(e^{-\Gamma_L t} + e^{-\Gamma_H t}) + |A_s \bar{A}_s| \cos(\delta_s + \gamma)(e^{-\Gamma_L t} - e^{-\Gamma_H t}) . \end{aligned} \quad (10)$$

The untagged decay rate into $D_s^+ K^-$ has a similar expression, in which $\cos(\delta_s + \gamma)$ is replaced by $\cos(\delta_s - \gamma)$. In order to extract both $\cos(\delta_s + \gamma)$ and $\cos(\delta_s - \gamma)$, thus eliminating part of the discrete ambiguity in γ , one needs independent information about $|A_s|^2$. This information can be obtained from the flavor tagged rates, or by relating $|A_s|^2$ through factorization to the measured value of the CKM-favored rate $|A(B_s \rightarrow D_s^- \pi^+)|^2$.

3 α from $B \rightarrow \pi\pi$

In the CKM framework $\alpha = \pi - \beta - \gamma$. This phase occurs in the time-dependent rate of $B^0(t) \rightarrow \pi^+ \pi^-$ and would dominate its asymmetry if only one amplitude (“tree” T) contributes. In reality this process involves a second amplitude (P) due to penguin operators which carries a different weak phase than the dominant tree amplitude. This leads to a more general form of the time-dependent asymmetry, which includes in addition to the $\sin \Delta m t$ term a $\cos(\Delta m t)$ term due to direct CP violation [10]

$$\mathcal{A}(t) = a_{\text{dir}} \cos(\Delta m t) + \sqrt{1 - a_{\text{dir}}^2} \sin 2(\alpha + \theta) \sin(\Delta m t) . \quad (11)$$

Both a_{dir} and θ are given roughly by the ratio of penguin to tree amplitudes, $a_{\text{dir}} \sim 2|P/T| \sin \delta_{\pi\pi}$, $\theta \sim |P/T| \cos \delta_{\pi\pi}$, where $\delta_{\pi\pi}$ is an unknown strong phase. This measurement provides two equations for $|P/T|$, $\delta_{\pi\pi}$ and α , which is insufficient for measuring α . A rough estimate of $|P/T|$, based on CKM and QCD factors, yielded some time ago the value 0.1 [32]. When flavor SU(3) is applied to relate penguin and tree amplitudes in recently measured $B \rightarrow \pi\pi$ and $B \rightarrow K\pi$ [33] one finds [34] $|P/T| = 0.3 \pm 0.1$. As mentioned, precise knowledge of this ratio could provide very useful information about α [10, 35]. Calculations of $|P/T|$ [12, 36] involve systematic theoretical errors which are uncontrollable at present.

One way of eliminating the penguin effect is by measuring also the time-integrated rates of $B^0 \rightarrow \pi^0\pi^0$, $B^+ \rightarrow \pi^+\pi^0$ and their charge-conjugates [37]. The three $B \rightarrow \pi\pi$ amplitudes obey an isospin triangle relation,

$$A(B^0 \rightarrow \pi^+\pi^-)/\sqrt{2} + A(B^0 \rightarrow \pi^0\pi^0) = A(B^+ \rightarrow \pi^+\pi^0) , \quad (12)$$

and a similar relation holds for the charge-conjugate processes. One uses the different isospin properties of the penguin ($\Delta I = 1/2$) and tree ($\Delta I = 1/2, 3/2$) contributions and the well-defined weak phase (γ) of the tree amplitude. By constructing the two isospin triangles one may measure the correction to $\sin 2\alpha$ in the second term of the asymmetry in Eq. (11)

An electroweak penguin contribution could spoil this method [38] since it involves a $\Delta I = 3/2$ component. This implies that the amplitudes of $B^+ \rightarrow \pi^+\pi^0$ and its charge-conjugate differ in phase, which introduces a correction at the level of a few percent in the isospin analysis. However, even this small correction can be taken into account analytically in the isospin analysis [39]. Other corrections, from isospin breaking in $\pi^0 - \eta$ mixing [40], turn out to be small for large values of $|P/T|$.

The difficulty of measuring α without knowing $|P/T|$ seems to be experimental rather than theoretical. The average branching ratios obtained from three experiments [33] $\mathcal{B}(B^0 \rightarrow \pi^+\pi^-) = (5.6 \pm 1.3) \times 10^{-6}$, $\mathcal{B}(B^+ \rightarrow \pi^+\pi^0) = (4.6 \pm 2.0) \times 10^{-6}$, are somewhat lower than anticipated some time ago [41]. The branching ratio into two neutral pions is likely to be smaller, since it obtains only contributions from penguin and color-suppressed amplitudes. In the most optimistic case, when these contributions interfere constructively, this branching ratio could lie just below the branching ratios measured for charged pions. A small $B^0 \rightarrow \pi^0\pi^0$ branching ratio and an experimentally

indistinguishable background may cause serious difficulties in applying this method. One will have to wait a while before measuring this branching ratio with sufficient precision. Stringent limits on this branching ratio would impose an interesting bound on the uncertainty in $\sin(2\alpha)$ obtained from the asymmetry in $B^0(t) \rightarrow \pi^+\pi^-$ [42]

$$\sin(\delta\alpha) \leq \sqrt{\frac{\mathcal{B}(B \rightarrow \pi^0\pi^0)}{\mathcal{B}(B^\pm \rightarrow \pi^\pm\pi^0)}} \quad (13)$$

Other ways of constraining this uncertainty were discussed in [43].

The isospin method for resolving penguin pollution in $\sin 2\alpha$ can also be applied to $B \rightarrow \rho\pi$ decays [44], of which there exist five charge modes some of which have already been measured [33], or to $B \rightarrow a_0(980)\pi$ [45]. Studying Dalitz plots of $B \rightarrow 3\pi$ [46], in which amplitudes describing different resonance bands involve unknown relative phases and interfere with an unknown three pion nonresonant amplitude, may be quite challenging [47].

4 γ from $B \rightarrow PP$

4.1 Flavor SU(3) relates $B/B_s \rightarrow \pi\pi, K\pi, K\bar{K}$

A large number of charmless B and B_s decays to two light pseudoscalars can be related to each other under approximate flavor SU(3) symmetry. This program started quite a few years ago [48] as a way of classifying hadronic weak amplitudes in terms of quark diagrams, and has been applied extensively for the past seven years to the $\Delta B = 1$, $\Delta C = 0$ low energy effective Hamiltonian for the specific purpose of determining weak phases. Whereas the first few attempts neglected second order electroweak penguin (EWP) contributions [49, 50], a large variety of proposals [51] were made after noting [52] the importance of these terms. Here we will review briefly the most common features of these proposals, focusing on one particular result.

The low energy effective weak Hamiltonian describing $\Delta S = 1$ charmless B decays, such as $B \rightarrow K\pi$, is [53]

$$\mathcal{H}_{\text{eff}}^{(s)} = \frac{G_F}{\sqrt{2}} \left[V_{ub}^* V_{us} \left(\sum_1^2 c_i Q_i^{us} + \sum_3^{10} c_i Q_i^s \right) + V_{cb}^* V_{cs} \left(\sum_1^2 c_i Q_i^{cs} + \sum_3^{10} c_i Q_i^s \right) \right] , \quad (14)$$

where c_i are scale-dependent Wilson coefficients and the flavor structure of the various four-quark operators is $Q_{1,2}^{qs} \sim \bar{b}q\bar{q}s$, $Q_{3,\dots,6}^s \sim \bar{b}s \sum \bar{q}'q'$, $Q_{7,\dots,10}^s \sim \bar{b}s \sum e_{q'} \bar{q}'q'$ ($q' = u, d, s, c$). In the $\Delta S = 0$ Hamiltonian describing $B \rightarrow \pi\pi$ one must replace $s \rightarrow d$. The ten operators consist of two $(V - A)(V - A)$ current-current operators ($Q_{1,2}$), four QCD penguin operators ($Q_{3,4,5,6}$), and four EWP operators ($Q_{7,8,9,10}$) with different chiral structures. One makes use of their following two properties:

- All four-quark operators, $(\bar{b}q_1)(\bar{q}_2q_3)$, can be decomposed into a sum of $\overline{\mathbf{15}}$, $\mathbf{6}$ and $\overline{\mathbf{3}}$ representations [48].
- The EWP operators with dominant Wilson coefficients, Q_9 and Q_{10} , have a $(V - A)(V - A)$ structure, and their components transforming as given SU(3) representations are proportional to the corresponding components of the current-current operators [54].

All $B/B_s \rightarrow PP$ decays (where final states belong to $\mathbf{1}$, $\mathbf{8}$ and $\mathbf{27}$) can then be expressed in terms of five SU(3) reduced amplitudes, or alternatively in terms of five independent combinations of eight diagrams [49]: Tree (T), Color-suppressed (C), Penguin (P), Annihilation (A), Exchange (E), Penguin Annihilation (PA), EWP (P_{EW}) and Color-suppressed EWP (P_{EW}^c). A useful proportionality relation between EWP and current-current (“tree”) operators is

$$\mathcal{H}_{EWP}^{(q)}(\overline{\mathbf{15}}) = -\frac{3}{2} \frac{c_9 + c_{10}}{c_1 + c_2} \frac{V_{tb}^* V_{tq}}{V_{ub}^* V_{uq}} \mathcal{H}_T^{(q)}(\overline{\mathbf{15}}), \quad q = s, d, \quad (15)$$

where $(c_9 + c_{10})/(c_1 + c_2) \approx -1.12\alpha$. This relation between EWP and tree amplitudes simplifies the analysis in certain cases.

Consider, for instance, $B \rightarrow (K\pi)_{I=3/2}$, where $|I = 3/2\rangle = |K^0\pi^+\rangle + \sqrt{2}|K^+\pi^0\rangle$. This process obtains only contributions from $\overline{\mathbf{15}}$ operators, and therefore its EWP and current-current amplitudes are proportional to each other and involve a common strong phase. Their ratio is given by

$$-\delta_{EW} e^{-i\gamma} = -\frac{3}{2} \frac{c_9 + c_{10}}{c_1 + c_2} \frac{V_{tb}^* V_{ts}}{V_{ub}^* V_{us}} = -(0.65 \pm 0.15) e^{-i\gamma}. \quad (16)$$

This feature can be used to obtain a bound on γ in $B^\pm \rightarrow K\pi$ [55]. The result can be summarized as follows. Defining a charge-averaged ratio of

rates

$$R_*^{-1} \equiv \frac{2[B(B^+ \rightarrow K^+\pi^0) + B(B^- \rightarrow K^-\pi^0)]}{B(B^+ \rightarrow K^0\pi^+) + B(B^- \rightarrow \bar{K}^0\pi^-)} , \quad (17)$$

one derives the following inequality, to leading order in small quantities

$$|\cos \gamma - \delta_{EW}| \geq \frac{|1 - R_*^{-1}|}{2\epsilon} , \quad (18)$$

where [33, 50]

$$\epsilon = \frac{|V_{ub}^* V_{us}|}{|V_{tb}^* V_{ts}|} \frac{|T + C|}{|P + EW|} = \sqrt{2} \frac{V_{us}}{V_{ud}} \frac{f_K}{f_\pi} \frac{|A(B^+ \rightarrow \pi^0\pi^+)|}{|A(B^+ \rightarrow K^0\pi^+)|} = 0.20 \pm 0.05 . \quad (19)$$

In the above isospin symmetry relates the dominant Penguin amplitudes in $B^+ \rightarrow K^+\pi^0$ and $B^+ \rightarrow K^0\pi^+$. SU(3) is used in the ratio (16) of subdominant EWP and tree amplitudes, and SU(3) breaking is introduced through f_K/f_π in (19) when evaluating the tree amplitude in $B^+ \rightarrow K^+\pi^0$.

A useful constraint on γ follows for $R_*^{-1} \neq 1$. The error of the present average value [33], $R_*^{-1} = 1.45 \pm 0.46$, ought to be reduced before drawing firm conclusions. Further information about γ , applying also to the case $R_*^{-1} = 1$, can be obtained by measuring separately B^+ and B^- decay rates [56]. The solution obtained for γ involves uncertainties due to SU(3) breaking in subdominant amplitudes and an uncertainty in $|V_{ub}/V_{cb}|$, both of which affect the value of δ_{EW} . Combined with errors in $\epsilon \propto |A(B^+ \rightarrow \pi^+\pi^0)/A(B^+ \rightarrow K^0\pi^+)|$, and in rescattering effects to be discussed below, the resulting uncertainty in γ is unlikely to be smaller than 10 or 20 degrees [56].

4.2 U-spin in charmless B decays

A subgroup of flavor SU(3), discrete U-spin symmetry interchanging d and s quarks, plays a particularly interesting and quite general role in charmless B decays [57]. Consider the effective Hamiltonian in Eq. (14). Each of the four-quark operators represents an s component (“down”) of a U-spin doublet, so that one can write in short

$$\mathcal{H}_{\text{eff}} = V_{ub}^* V_{us} U^s + V_{cb}^* V_{cs} C^s , \quad (20)$$

where U and C are U-spin doublet operators. Similarly, the effective Hamiltonian responsible for $\Delta S = 0$ decays involves d components (“up” in U-spin)

of corresponding operators multiplying CKM factors $V_{ub}^* V_{ud}$ and $V_{cb}^* V_{cd}$,

$$\mathcal{H}_{\text{eff}} = V_{ub}^* V_{ud} U^d + V_{cb}^* V_{cd} C^d . \quad (21)$$

The structure of the Hamiltonian implies a general relation between two decay processes, $\Delta S = 1$ and $\Delta S = 0$, in which initial and final states are obtained from each other by a U-spin transformation, $U : d \leftrightarrow s$. Writing the $\Delta S = 1$ amplitude as

$$A(B \rightarrow f, \Delta S = 1) = V_{ub}^* V_{us} A_u + V_{cb}^* V_{cs} A_c , \quad (22)$$

the corresponding $\Delta S = 0$ amplitude is given by

$$A(UB \rightarrow Uf, \Delta S = 0) = V_{ub}^* V_{ud} A_u + V_{cb}^* V_{cd} A_c . \quad (23)$$

Here A_u and A_c are complex amplitudes involving CP-conserving phases. The amplitudes of the corresponding charge-conjugate processes are

$$A(\bar{B} \rightarrow \bar{f}, \Delta S = -1) = V_{ub} V_{us}^* A_u + V_{cb} V_{cs}^* A_c , \quad (24)$$

and

$$A(U\bar{B} \rightarrow U\bar{f}, \Delta S = 0) = V_{ub} V_{ud}^* A_u + V_{cb} V_{cd}^* A_c . \quad (25)$$

Unitarity of the CKM matrix [58], $\text{Im}(V_{ub}^* V_{us} V_{cb} V_{cs}^*) = -\text{Im}(V_{ub}^* V_{ud} V_{cb} V_{cd}^*)$, implies, denoting CP rate differences by Δ ,

$$\begin{aligned} \Delta(B \rightarrow f) &\equiv \Gamma(B \rightarrow f) - \Gamma(\bar{B} \rightarrow \bar{f}) = \\ -\Delta(UB \rightarrow Uf) &\equiv -[\Gamma(UB \rightarrow Uf) - \Gamma(U\bar{B} \rightarrow U\bar{f})] . \end{aligned} \quad (26)$$

Namely, *CP rate differences in decays which go into one another under interchanging s and d quarks have equal magnitudes and opposite signs*. This rather powerful result, following from U-spin within the CKM framework, can be demonstrated in numerous decay processes, including two body, quasi-two body, and multibody hadronic and radiative B decays. In view of this result, it is rather easy to look for physics beyond the standard model in pairs of U-spin related processes. Since it seems unlikely that strong phases change sign under U-spin breaking, measuring asymmetries with equal signs in such a pair would be a clear signal for new physics.

Let us focus our attention on six U-spin related pairs of processes (out of a total of sixteen decays) of the type $B, B_s \rightarrow \pi\pi, K\pi, K\bar{K}$:

1. $B^0 \rightarrow K^+\pi^-$ vs. $B_s \rightarrow \pi^+K^-$, 2. $B_s \rightarrow K^+K^-$ vs. $B^0 \rightarrow \pi^+\pi^-$,
3. $B^0 \rightarrow K^0\pi^0$ vs. $B_s \rightarrow \bar{K}^0\pi^0$, 4. $B^+ \rightarrow K^0\pi^+$ vs. $B^+ \rightarrow \bar{K}^0K^+$,
5. $B_s \rightarrow K^0\bar{K}^0$ vs. $B^0 \rightarrow \bar{K}^0K^0$, 6. $B_s \rightarrow \pi^+\pi^-$ vs. $B^0 \rightarrow K^+K^-$.

Equalities of CP rate-differences within each of these pairs can be used to test the validity of U-spin symmetry.

The first five pairs of processes are dominated by a large penguin amplitude P , such that the corresponding branching ratios are of order 10^{-5} . The amplitudes of the last pair involve the combination $PA + E$, which is expected to be very small [49] unless amplified by rescattering [59]. Neglecting rescattering, one estimates $\mathcal{B}(B^0 \rightarrow K^+K^-) \sim \mathcal{O}(10^{-7} - 10^{-8})$ [13]. To reach this level, the present experimental upper limit [33], $\mathcal{B}(B^0 \rightarrow K^+K^-) < 1.9 \times 10^{-6}$, should be improved by one or two orders of magnitude. Assuming that $PA + E$ can be neglected relative to P , one has

$$A(B_s \rightarrow K^+K^-) \approx A(B^0 \rightarrow K^+\pi^-) , \quad A(B_s \rightarrow \pi^+K^-) \approx A(B^0 \rightarrow \pi^+\pi^-) . \quad (27)$$

In the approximation of factorized hadronic amplitudes [12, 13], U-spin breaking is introduced through the ratio of corresponding form factors,

$$\begin{aligned} A(B_s \rightarrow K^+K^-)/A(B^0 \rightarrow K^+\pi^-) &= A(B_s \rightarrow K^-\pi^+)/A(B^0 \rightarrow \pi^+\pi^-) \\ &= F_{B_s K}(m_K^2)/F_{B\pi}(m_K^2) \approx F_{B_s K}(m_\pi^2)/F_{B\pi}(m_\pi^2) . \end{aligned} \quad (28)$$

The approximate equality in ratios of form factors holds to within 1%. *The rates of these four processes can be used not only to determine the U-spin breaking factor in the ratio of amplitudes, but also to check the factorization assumption by finding equal ratios of amplitudes in the two cases.*

4.3 γ from $B/B_s \rightarrow K\pi$

The four processes appearing in (27) play a useful role in determining γ . Here we describe a scheme based on self-tagging $K\pi$ decays of B^0 and B_s mesons [60]. A complementary method, based on time-dependent flavor-tagged $B^0(t) \rightarrow \pi^+\pi^-$ and $B_s(t) \rightarrow K^+K^-$ [61], may reach a similar precision.

Writing the amplitudes for $B^0 \rightarrow K^+\pi^-$ and $B_s \rightarrow K^-\pi^+$ as in Eqs. (22) and (23), respectively, we note that the rates for these processes and their charge-conjugates depend on four quantities, $|V_{ub}^*V_{us}A_u|$, $|V_{cb}^*V_{cs}A_c|$, $\delta_{K\pi} \equiv \text{Arg}(A_uA_c^*)$ and $\gamma \equiv \text{Arg}(-V_{ub}^*V_{ud}V_{cb}V_{cd}^*)$. Because of the equality of CP rate-differences in the two processes, a determination of γ requires another input. This input is provided by $|A(B^+ \rightarrow K^0\pi^+)| = |V_{cb}^*V_{cs}A_c|$, where small rescattering corrections are neglected as argued above.

Defining two charge-averaged ratios of rates

$$R \equiv \frac{\Gamma(B^0 \rightarrow K^\pm\pi^\mp)}{\Gamma(B^\pm \rightarrow K\pi^\pm)} , \quad R_s \equiv \frac{\Gamma(B_s \rightarrow K^\pm\pi^\mp)}{\Gamma(B^\pm \rightarrow K\pi^\pm)} , \quad (29)$$

and CP violating pseudo-asymmetries

$$\mathcal{A}_0 \equiv \frac{\Delta(B^0 \rightarrow K^\pm\pi^\mp)}{\Gamma(B^\pm \rightarrow K\pi^\pm)} , \quad \mathcal{A}_s \equiv \frac{\Delta(B_s \rightarrow K^\pm\pi^\mp)}{\Gamma(B^\pm \rightarrow K\pi^\pm)} , \quad (30)$$

one finds

$$R = 1 + r^2 + 2r \cos \delta_{K\pi} \cos \gamma , \quad (31)$$

$$R_s = \tan^2 \theta_c + (r/\tan \theta_c)^2 - 2r \cos \delta_{K\pi} \cos \gamma , \quad (32)$$

$$A_0 = -A_s = -2r \sin \delta_{K\pi} \sin \gamma , \quad (33)$$

where $r \equiv |V_{ub}^*V_{us}A_u|/|V_{cb}^*V_{cs}A_c|$. SU(3) breaking can be checked in (33) and used for improving the precision in γ obtained from these four quantities. It is estimated [60] that a precision of 10° in γ can be achieved in experiments to be performed at the Fermilab Tevatron Run II program [62].

5 Conclusions

Measuring CP asymmetries in B decays is important for several reasons:

- CP violation should be observed outside the K system ($B^0 \rightarrow J/\psi K_S$).
- In many cases relative signs of asymmetries test the CKM picture (U-spin related processes).
- Certain asymmetries are predicted by CKM to be very small ($B_s \rightarrow J/\psi\phi$). Sizable asymmetries are signals of new physics.

- Asymmetries in different processes test and overconstrain the CKM parameters. In certain cases phase determinations are theoretically clean ($B^0 \rightarrow J/\psi K_S$, $B \rightarrow DK$, $B_s \rightarrow D_s K$), some are difficult ($B^0 \rightarrow \pi^0 \pi^0$), and others still involve theoretical uncertainties due to rescattering and SU(3) breaking effects ($B/B_s \rightarrow PP$). There are ways to measure and set bounds on these corrections. A combined experimental and theoretical effort should resolve these uncertainties.
- CP asymmetries will make us happy and our work interesting in the next few years.

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